# Modelling, Simulating, and Visualizing Granular Materials

Arash Habibi, Annie Luciani, Emmanuel Manzotti

# Modelling, Simulating and Visualizing Granular Materials

A. Habibi, A. Luciani, E. Manzotti
ACROE - LIFIA
INPG - 46 avenue Félix Viallet
38 031 Grenoble cedex
FRANCE
Tél : (33) 76 57 46 69
Fax : (33) 76 57 46 02
e-mail : <luciani@imag.fr>

## Abstract

The shape and the motion of dunes are most impressive and beautiful scenes. Granular materials in general and sand in particular display a variety of behaviours, that are in many ways different from those of either liquids or solids. These behaviours are very partially understood. The objective of this work is, on the one hand, the understanding of the macroscopic behaviour of this type of material, and on the other hand, the achievement of animation sequences for artistic purposes. The grain models described here were implemented and simulated according to the *Cordis-Anima* formalism. This means that physical objects in general, and granular material in particular, are considered as a set of punctual masses linked together by linear or non-linear spring-damper interactions. These granular objects were visualized thanks to the *engraved screen* algorithm : this "screen" is a physically simulated carving plate acting as a visualization support.

The models and simulations produced here account for complex phenomena such as the constitution of a pile from independant particles, as well as avalanch and internal collapse phenomena.

## I. Introduction

Sugar, flour, or sand, are examples of familiar granular materials. They are also frequently handled in industries such as pharmaceuticals industry and the construction of highways and dams. In spite of all this, and in spite of the beautiful shapes and movements produced by these objects, their dynamic behaviour is not very well understood.

This work aims at the elaboration of physical models of non-structured objects such as sandpiles and the achievement of dynamic simulations.

We have two specific objectives : one concerns the synthesis of beautiful animation sequences. The other concerns the knowledge of the properties of granular materials by physical simulation.

Considered as an image synthesis project, the most attractive in this work, is the beauty of the shapes and movements produced by these materials, all the more as no synthesis animation of this type of object has been produced yet. The most fascinating is precisely the organic link between the created shape that can be static, and the movement of the object that leads to shapes that can only be qualified paradoxically as fluid.

On the other hand, considered as a project on physical modelling and simulation, the most attractive is the the lack of dynamic knowledge available on granular materials. These objects can be classified as "non-structured" objects.

Our approach consists of determining the minimal conditions required for the formation of a pile, for the occurrence of avalanches, collapses, arching etc. When these minimal conditions are determined, more complex models can be thought of. But this goes beyond the scope of this first paper.

In section II, we expose a certain number of very special mechanical properties that show the specificities of granular materials. In section III we expose different ways in which the scientific community has tackled the study of granular materials. In section IV, we present our model and the corresponding results. In section V, we present the way in which these materials were visualized. The work on the physical modelling of the granular materials was carried out by Annie Luciani and Emmanuel Manzotti. The work on the visualization of this type of materials was carried out by Annie Luciani and Arash Habibi.

## II. Granular materials : special properties

Sand and other granular materials have very particular properties. They are composed of solid particles. However, when they are studied from a macroscopic point of view, they cannot be easily classified as either solids or liquids. For example, they have no shape of their own. Their overall shape is usually the shape of their container. They can also flow out of their container through a hole as do liquids. Still, they have several particular properties that can definitely not be explained by classical fluid mechanics.

**a.** As it is explained in [2], the pressure at the bottom of a sufficiently tall container, filled with sand up to a height $h$ is independant of $h$. For a liquid, pressure is proportional to h. This is because the friction of the particles along the wall of the container is sufficient to withstand the weight of the extra mass placed on its top.

**b.** In a gas or in a liquid at finite temperatures, pressure is a continuous isotropic function of space because of the thermal motion. Conversely there is almost no thermal motion in granular materials Therefore granular materials are inherently inhomogeneous and the force network providing the stability of the system is nonuniform. Lines conveying high constraints surround regions where pressure is almost zero. (See [2] and [4])

**c.** When sand is poured on a flat surface, it forms a pile whose features (for example the slope angle) depend on the physical properties of the particles in a way that is still to be determined. Of course, this phenomenon cannot be observed for liquids.

**d.** In certain conditions, when sand continues pouring on the pile, avalanches occur at the surface of the pile. This is also a phenomenon that needs to be explored.

**e.** Granular materials show a phenomenon known as *arching*. In a random configuration of grains, there will be places where arches appear naturally, leaving empty regions below (See also [4] and [6])

**f.** In some conditions (yet to be specified) a sand pile can collapse : At the breaking of the force lines mentionned in b, inner collapses occur, in which entire blocks move.

These are specific properties of granular materials that differ from the properties of fluids. In order to account for these properties, plain fluid mechanics is not sufficient : other types of model are needed.

## III. Granular materials : special models

The various approaches can be classified as granular or non-granular.

### 3.1. Non-granular models

The non-granular approaches use emergent properties i.e. properties that are not inherent in one particle but inherent in a large group of particles. In some cases, this leads to continuous models where the granular object is considered as a homogeneous body. In other cases, the object is discrete but the simulation deals with global emergent magnitudes such as slope or statistical magnitudes such as temperature.

Xin Li and J.M. Moshell [1] have modelled avalanches in a sandpile by considering that a pile can be divided into homogeneous continuous vertical slices. The sand on the top of each slice can slide to a neighbouring slice with friction forces expressed by the Coulomb model, as well as cohesion forces and interlocking forces between particles.

But it is also very tempting to use the models of statistical mechanics. The random component of the movement of flowing grains in an avalanche has been compared with thermal motion. This has lead to the definition of a "granular temperature" for the study of avalanches. A. Mehta and S.F. Edwards [3] have proposed another analogy with statistical mechanics. They have considered the volume of a granular body as an energy to be minimized. Consequently the number of different configurations compatible with this volume defines entropy and it follows an entity that plays the same role as temperature does in fluid materials.

Others have elaborated an analogy between avalanches in sandpiles and second order transitions. This has lead to the notion of *self-organized criticality* [2].The slope of a sandpile is compared with temperature in second order transitions. A system in the neighbourhood of the critical temperature is characterized by both long-range temporal and spatial correlations. Similarly, in the neighbourhood of the critical slope, small perturbations lead to long-range interactions that result in avalanches.

### 3.2. Granular approaches

The use of computers, and more specifically simulation, enables another approach in which granular material are explicitly considered as a set of discrete particles interacting locally with the neighbouring particles regardless of any consideration on slope or global temperature. Simulation enables to observe the collective behaviour of a large number of particles in terms of microscopic properties i.e. the properties of individual grains.

Geometrical simulations have lead to experimental results as for the compacity of granular materials and arching possibilties ([5] and [7]). But as the authors emphasize these simulations do not account for important dynamic phenomena such as avalanches and the fact that a given particle can move the particles on which it lies.

The work presented in this paper corresponds definitely to a granular approach. A granular material is explicitly modelled as a set of discrete masses in physical interaction. The parameters of the simulation are the physical characteristics of the individual grains. The simulations were achieved thanks to the *Cordis-Anima* modeller-simulator (See [7]). This means that matter is modelled as a set of punctual masses linked together by discrete spring-damper interactions.

## IV. A Sand Pile model

### IV.1. Experiments

Our aim is to construct a sand pile and to find out the necessary and sufficient conditions in which sand poured on the ground forms a pile.

We assumed that the formation of a pile can be caused by the following conditions. We tested which of these conditions are necessary and which are sufficient. We know that when all four are not fulfilled, no pile is formed.

i. Dry friction between particles (caused by the non-regular shape of the particle)
ii. Dry friction between particles (caused by the material the particles are made of)
iii. Dry friction between the particles and the ground
iv. Viscosity between particles.

We limited our tests to conditions i. iii. and iv. In other words, in our models, dry friction was only caused by the irregular shape of the particles.

The point is to test each single condition while the other two are not fulfilled (for example i. true, iii. false, iv. false). If no pile is formed in either case, then no single condition is sufficient. We would have to do the tests with several conditions fulfilled. (i. true, iii. true, iv. false) etc.

### IV.2. Results

The results of these experiments were quite surprising :

In all cases, if a set of physical particles is poured on a <u>rugged</u> ground (i.e. with dry friction with the particles) a pile is formed. This was carried out with smooth particles and it holds even when the particles are perfectly round and non-viscous. As a consequence, friction between the particles and the ground is a sufficient condition for the formation of sand piles.

In all cases if a set of physical particles is poured on a smooth ground (i.e. with no friction with the particles) no pile is formed. This holds even for viscous particles with complex shapes. This means that friction between the particles and the ground is also a necessary condition for the formation of a pile.

Finally we end up with a very simple model : round, smooth, non-viscous, non-cohesive particles with simple repulsive interactions poured on a rugged ground. But this very simple model accounts for very complex phenomena : we observed stable shapes i.e. piles and stable cavities inside the piles. Dynamically we observed avalanches and internal collapses revealing that the force distribution within the pile is basically heterogeneous. Pressure and constraints in these simulated piles propagate along specific lines, through specific grains as in real granular materials.

Different values of stiffness and different grain sizes lead to different repose slopes and different dynamic behaviours. But one important consequence of these experiments is that a necessary and sufficient condition for the formation of piles of granular materials is friction between the particles and the ground. This conclusion is all the more important that we do not know of any such result achieved in the past and that such a result could only be achieved by simulation. In the real world, such a condition cannot be tested since friction between particles and the ground cannot be eliminated.

Figure 1.a to 1.c show a set of 700 round, smooth, non-viscous physical particles falling on a rugged ground (composed of fixed particles of the same type) and forming a dynamic pile. Each point represents one particle. Despite the rather poor visualization method used in this figure, and despite the lack of motion, one can almost distinguish the discrete regions in which the pile is divided and that were mentioned above as inherent in granular materials. This division will be much clearly observed in the video demonstrations. Figures 2.a through 5.a illustrate the same type of particles dragged from left to right by simulated gusts of wind and stopped by simulated physical obstacles (not represented here). Behind these obstacles the particles form dunes of various shapes depending on the shape of the obstacle and the way the particles arrive on it.

All these experiments have been recorded on video and have been used in the audio-visual artistic piece "ESQUISSES" (Sketches) realized by the ACROE.

## V. The visualization of sand piles and dunes

In the field of image synthesis granular materials have produced beautiful animation sequences as those of "ESQUISSES". These simulations were visualized by the engraved screen method already described in our previous works [8]. This is a well-suited method for the visualization of very deformable objects such as smoke, fire, liquid surfaces and sand.

This method was inspired by the works of Alexander Alexeïev and Claire Parker who made and used the first real pin screen [9]. P.F. Lopes and M.R. Gomes [10] have also elaborated a pin screen model. However they deal basically with the optical properties of their model, whereas we are much more interested in the physical properties of the engraved screen.

The engraved screen is a physically simulated deformable surface composed of a great number of discrete mass elements (up to 400 000). The objects to be visualized are put against this deformable surface and act as chisels in a sort of dynamic bas-relief sculpture. If the screen has elastic properties, the traces disappear more or less rapidly according to the physical parameters of the screen. Furthermore, different values of cohesion between the masses of the screen produce surfaces that resemble rather a piece of wood or an elastic blanket.

A rather small number of grains (300 to 1500) seems to be sufficient for the simple observation of phenomena such as piling, avalanches and internal collapses. But for visualization, especially for artistic purposes, a far higher resolution is required.

One alternative would be to simulate a greater number of grains. This would result in a finer, more subtle simulation with more small scale phenomena. However, if this is not the major goal, i.e. if what is aimed at, is merely a visually acceptable visualization of practically the same phenomenon, the alternative would be interpolation or, what we usually call "clothing".

In this interpolation, the simulated grains would be the control points or the *skeletons*. Then interpolation would be carried out geometrically or physically between the grains.

In geometrical interpolation or clothing, the added points are defined by non-physical operations. This is the case of spline patches or implicit surfaces. In physical clothing, the additional points are mass elements physically linked to the skeletons. Thus the engraved screen is a means of physical interpolation.

A great number of results obtained by geometrical interpolation can be obtained by physical interpolation, provided that the interpolation points be sufficiently light and the interactions be sufficiently stiff and viscous. The converse is not true. For finite values of mass and interaction, the resulting interpolation becomes visually very complex, since it is the combination of the dynamics of both the skeleton and the clothing. Specific values of these parameters can result in the reinforcement of the visual perception of the movement or conversely inhibit this perception.

This is what we need for the visualization of sand piles and this is indeed what we used : Figures 2.b through 5.b correspond to the same simulations as those of figures 2.a through 5.a. The difference is the visualization method. The *a* figures are point visualizations whereas the *b* figures are engraved screen visualizations. Figures 2.b and 3.b correspond to geometrical clothings. Each pebble corresponds to one particle. No smoothing is performed in this case. The discrete nature of the screen is used in order to convey to each round particle more sharp shapes. Figures 4.b and 5.b correspond to physical clothings. The cohesion between each pin performs smoothing, which renders the fine sand effect. Moreover the viscous interaction between the pins and the ground causes each particle to leave a long trace behind in the course of its movement

In the resulting animation videos, we observed rich deformations and movements that cannot be obtained by any geometrical interpolation such as free-form surfaces. These demonstration videos will be shown during the presentation.

These results encourage us to go further. Current work aims at the modelling and the visualization of very deformable objects such as smoke and flames, with the same type of tools.

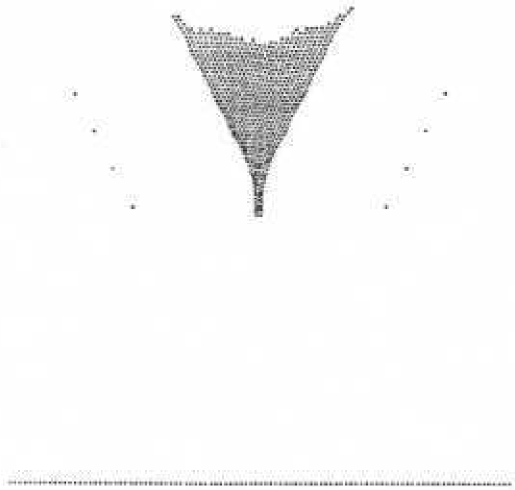

**Figure 1.a.** The sand in the hourglass...

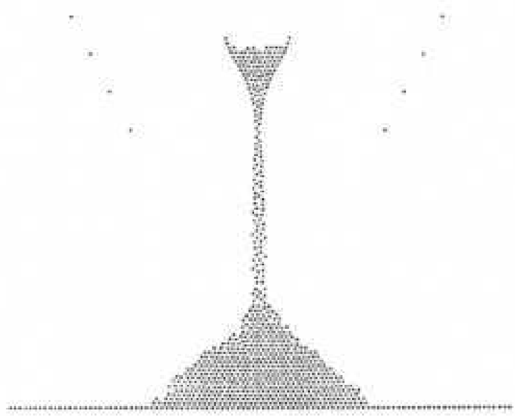

**Figure 1.b.** ... pouring down ...

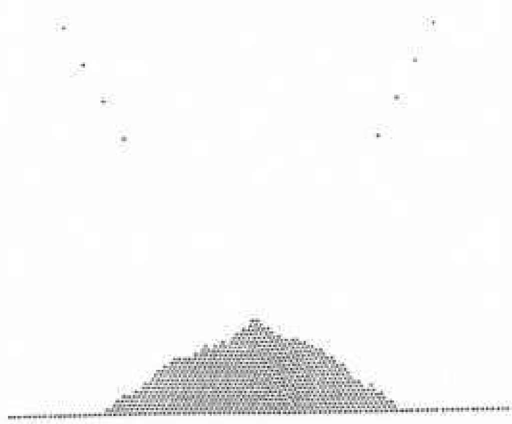

**Figure 1.c.** ... and forming a pile

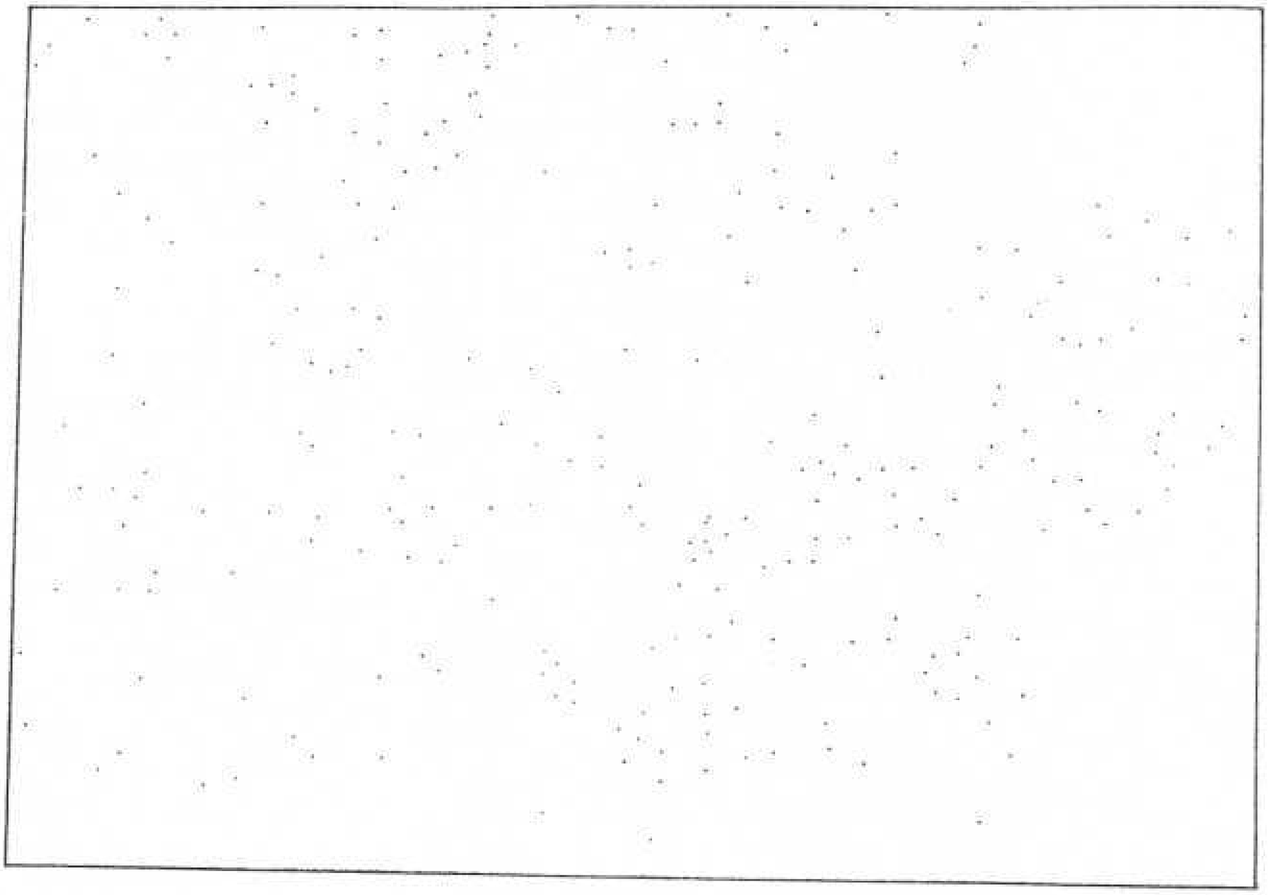

**Figure 2.a** Grains of sand carried by the wind

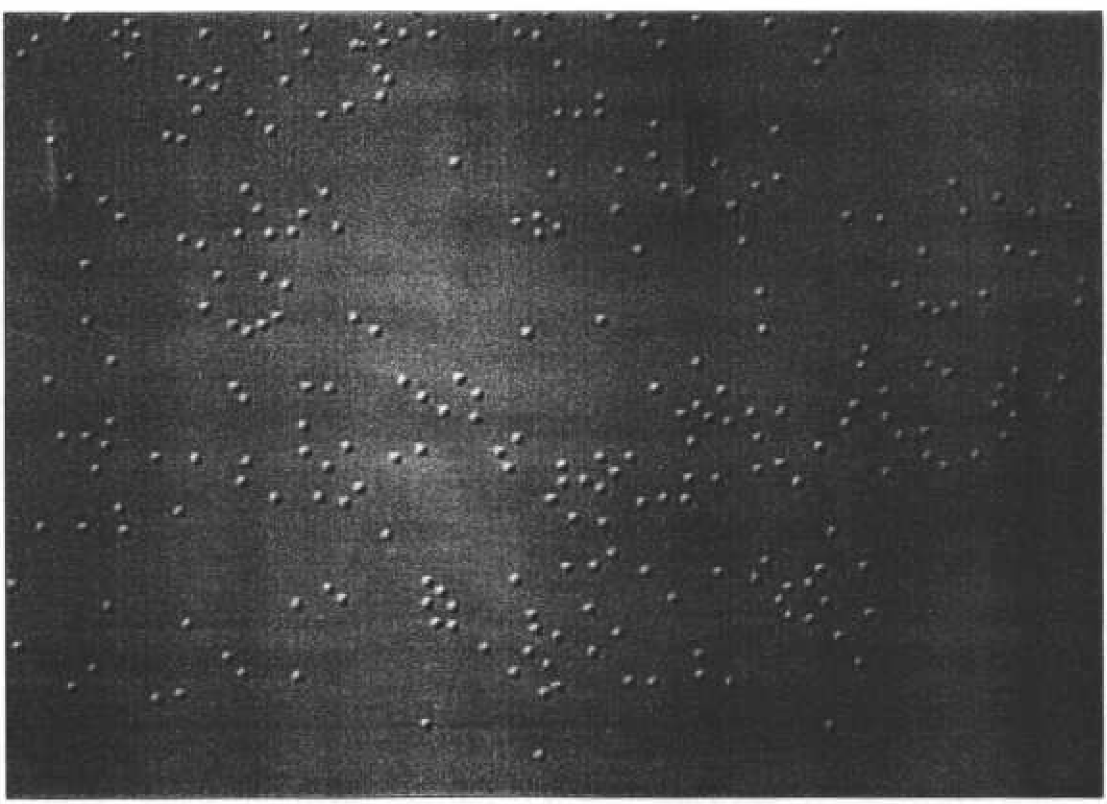

**Figure 2.b** The same figure with the engraved screen visualization

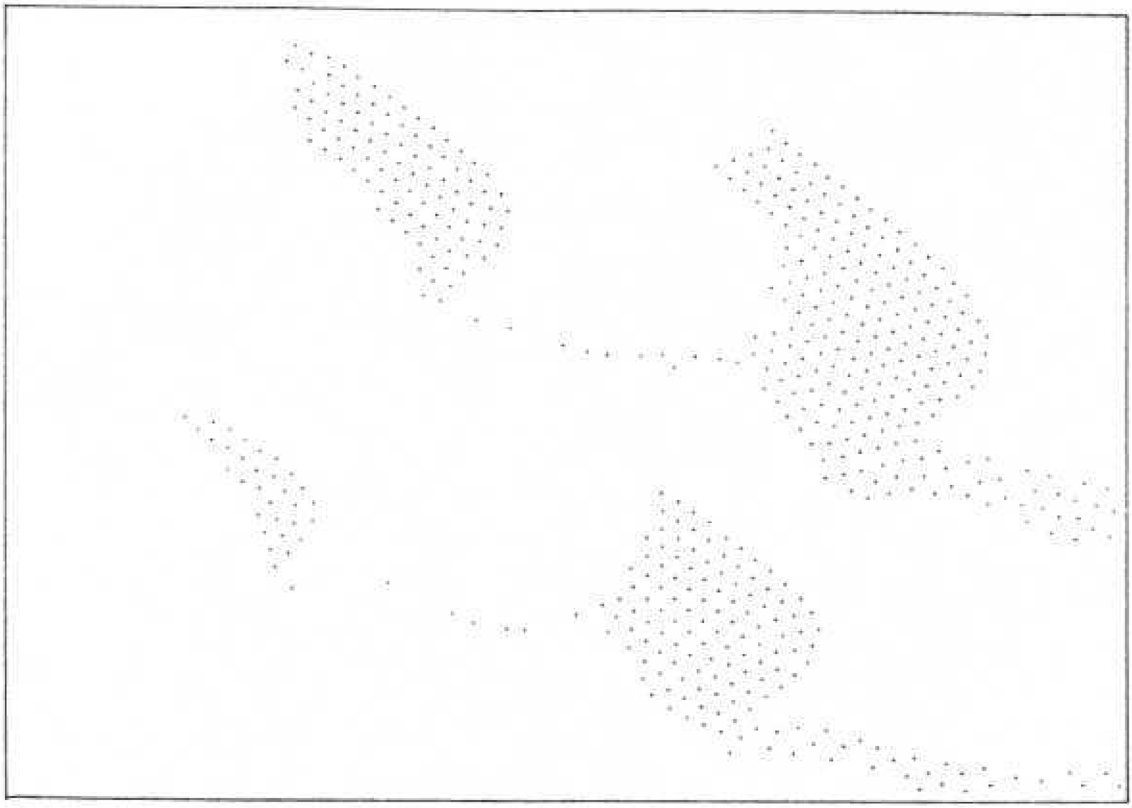

**Figure 3.a** Same sequence as in figure 2.a. The sand is stopped by four obstacles

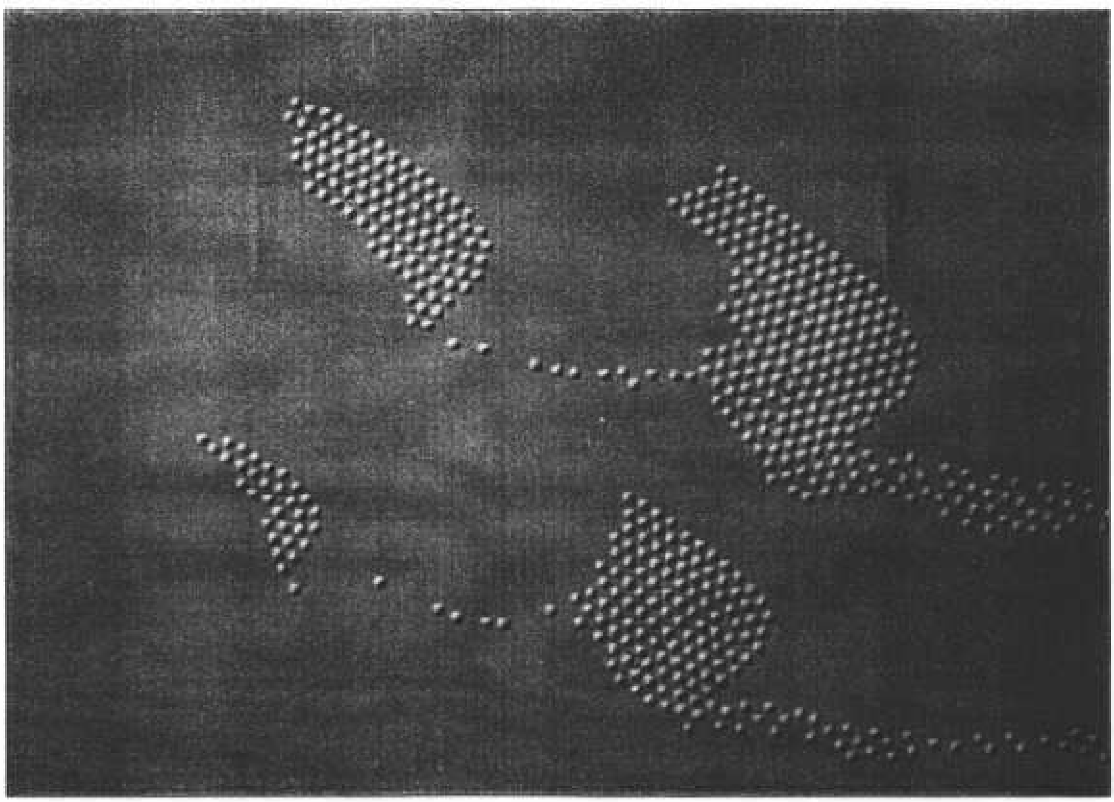

**Figure 3.b** The same figure with the engraved screen visualization

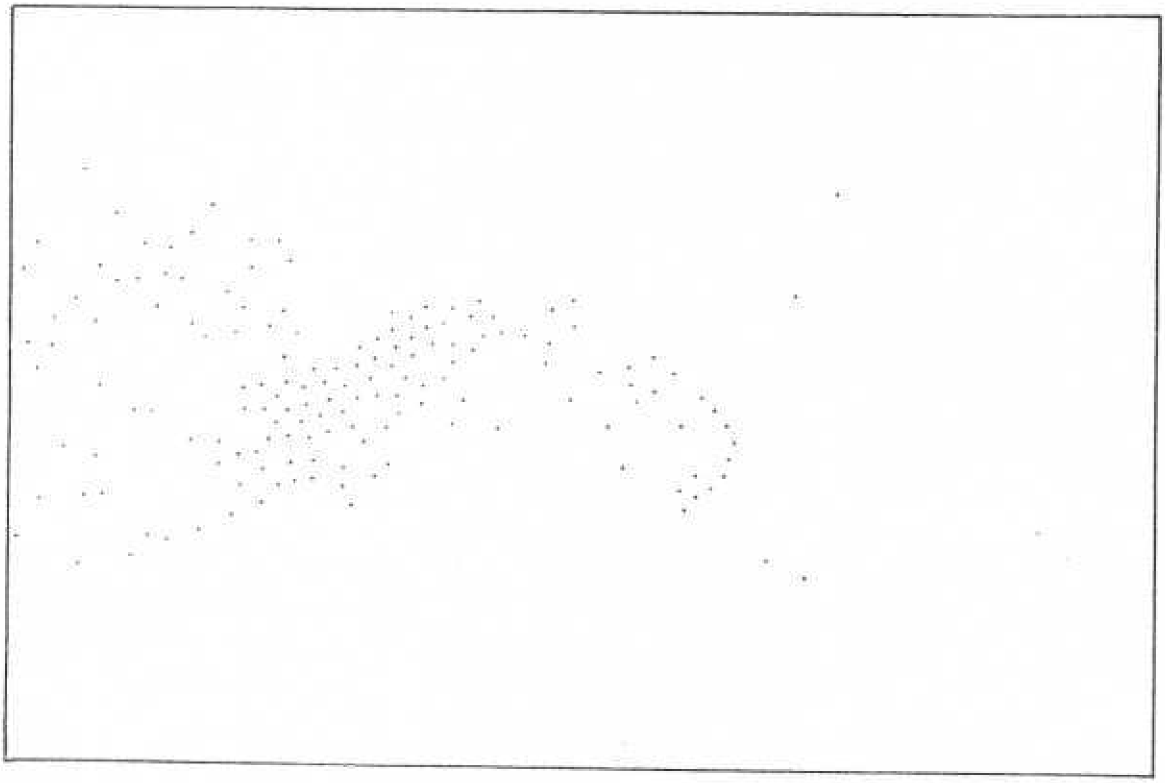

**Figure 4.a** Fine sand stopped by an obstacle and forming a dune

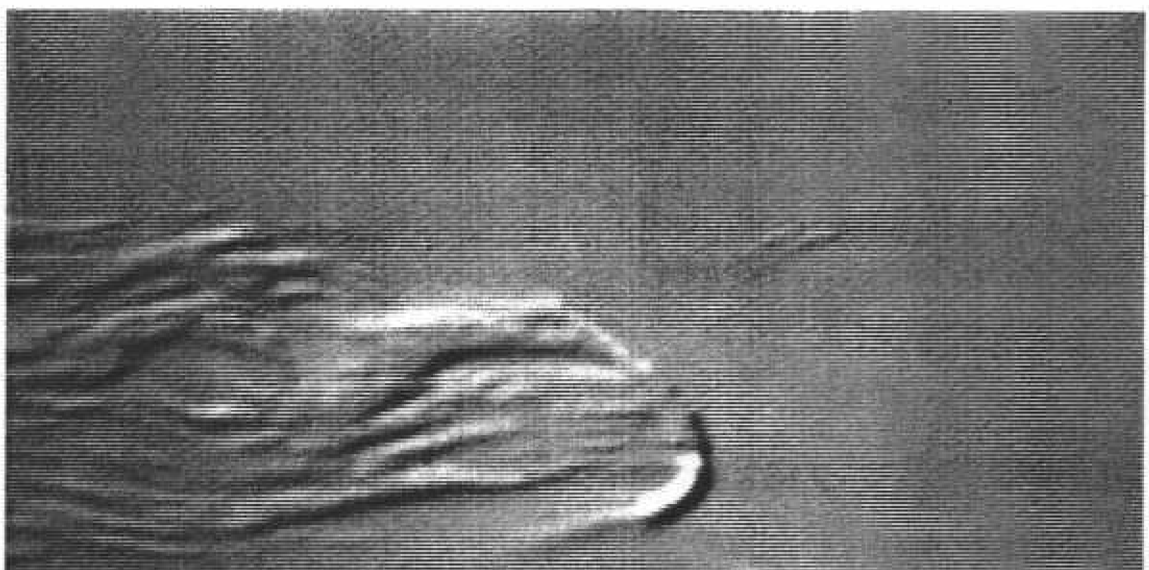

**Figure 4.b** The same figure with the engraved screen visualization
An instance of physical clothing

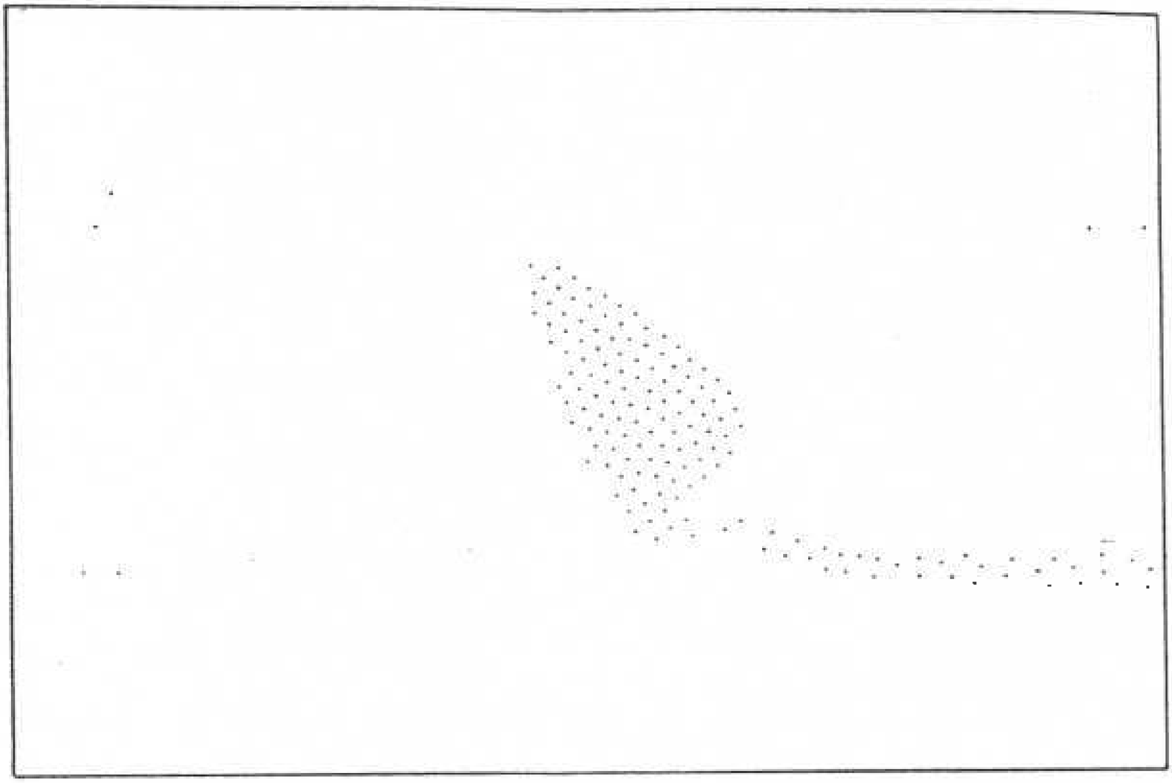

**Figure 5.a** Another moment in the same simulation

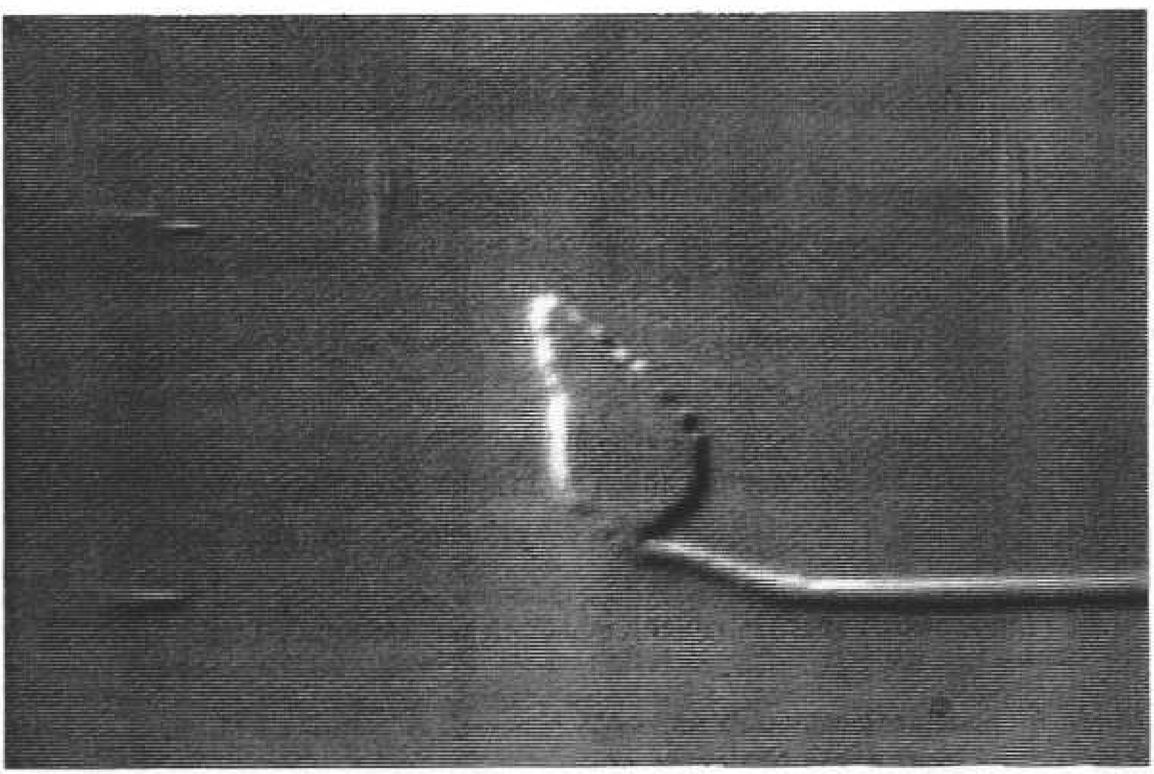

**Figure 5.b** The same figure with the engraved screen visualization

## References

1. Xin Li, J. Michael Moshell *Modeling Soil : Realtime dynamic Models for Soil Slippage and Manipulation* Proceedings of SIGGRAPH '93

2. Heinrich M. Jaeger, Sidney R. Nagel *Physics of the granular state* Science vol. 255, page 1523, 20 March 1992

3. Edwards S.F., Mehta A., *Dislocations in amorphous materials* Journal de Physique (Paris) vol. 50, page 2489, 15 September 1989. A. Mehta , S.F. Edwards, Phys. A, 157, 1091, 1989.

4. Ammi M., Bideau D., Troadec J.P., *Geometrical structure of disordered packings of regular polygons ; comparison with disc packings strucutres* Journal of Physics D, vol. 20, page 424, 14 April 1987 4.

5. Missiaen J. M. *Etude par analyse d'images des milieux granulaires : caractérisation des contacts et de l'homogénéité* PhD Thesis. Ecole Nationale Supérieure des Mines de Saint-Etienne (Saint-Etienne 1989)

6. Jullien R. *La ségrégation chez les grains de sable* La Recherche vol. 23 page 1198 (1992)

7. Cadoz C. Luciani A. Florens J.L. *CORDIS-ANIMA Système de modélisation et de simulation d'instruments et d'objets physiques pour la création musicale et l'image animée.* Proceedings of Colloque "Modèles physiques, Création musicale et Ordinateur" organized by the ACROE in 1990 Vol. 13, 14, 15, page 585. *La Maison des Sciences de l'Homme* editions March 1994.

8. Habibi A. Luciani A. *Physical models for the visualization of animated images "From the physical model to the eye"* Proceedings of the Fourth Eurographics Animation and Simulation Workshop" (Barcelona, Spain September 4-5 1993)

9. Salomon N. Just J. *Entretiens avec Alexandre Alexeïev et Claire Parker*, Annecy 1979

10. P.F. Lopes M.R. Gomes *A Computer Model for Pinscreen Simulation : A New Animation Pardigm*, Computer Graphics Forum, Volume 11, Number 1, March 1992, pp 31-42